Pergamon

www.elsevier.com/locate/jappmathmech

PII: S0021–8928(03)00073-X

J. Appl. Maths Mechs, Vol. 67, No. 3, pp. 453–460, 2003

0021–8928/$—see front matter

FEATURES OF THE REFLECTION OF WAVES WITH STRONG DISCONTINUITIES FROM THIN COMPRESSIBLE OBSTACLES†

V. A. ROMASHCHENKO

Kiev

e-mail: roma@ipp.adam.kiev.ua

(*Received* 1 *March* 2001)

Using the example of the plane contact problem of hydroelasticity theory, the multiple reflection of waves with strong discontinuities, propagating in an ideally elastic liquids, from thin films having a finite acoustic impedance, is investigated analytically. The wave solution is presented in the form of the sum of a basic component (no film) and a perturbed component. An algorithm is developed for the successive analytical calculation of the perturbed components after multiple reflections from an obstruction.

Using the example of the one-dimensional plane linear boundary-value problem of hydroelasticity, the multiple reflection of waves with strong discontinuities, propagating over a layer of liquid from ultrathin films was investigated in [1]; the film was modelled by an incompressible added mass, i.e. a body with an infinite acoustic impedance. Below, continuing these investigations, the problem is considered in a refined contact formulation, where the obstruction has a finite wave impedance and wave processes both through the thickness of the liquid and through the thickness of the film are considered.

Consider the contact one-dimensional wave problem of hydroelasticity. There is an infinite layer of ideally elastic liquid of thickness l and density ρ_1, and we will direct the x axis normal to the surface of the layer so that the line $x = 0$ corresponds to one surface and line $x = l$ corresponds to the opposite surface of the layer. A specified pressure pulse $f(t)$ in incident on the surface $x = 0$ (which we will call the left-hand surface), where f denotes the difference between the total and atmospheric pressure on the left-hand boundary of the liquid layer. The surface $x = l$ (which we will call the right-hand surface) is in contact, without detachment or leakage, with a thin infinite elastic solid sheet (a film) of thickness αl and density ρ_2. At the initial instant of time $t = 0$, the liquid and the obstacle are stationary and are under atmospheric pressure P_0 only. The earth's force of attraction will be neglected. The force excitation $f(t)$ is considered to be fairly small, and the problem is examined in the linear formulation for small displacements. The behaviour of the liquid will be described in Euler coordinates, and that of the solid in Lagrangian coordinates.

Using the velocity potential of the liquid ψ, we will formulate the corresponding contact hyperbolic boundary-value problem in the following way [1, 2]

$$\psi_{xx} = \frac{\psi_{tt}}{a_1^2}; \quad 0 \le x \le l, \quad t > 0; \quad u_{xx} = \frac{u_{tt}}{a_2^2}; \quad l \le x \le l + \alpha l, \quad t > 0$$

$$\psi = \psi_t = 0; \quad 0 \le x \le l, \quad t = 0; \quad u = u_t = 0; \quad l \le x \le l + \alpha l, \quad t = 0$$

$$\rho_1 \psi_t(0, t) = -f(t); \quad x = 0, \quad t > 0 \tag{1}$$

$$u_x(l + \alpha l, t) = 0; \quad x = l + \alpha l, \quad t > 0$$

$$\psi_x(l, t) = u_t(l, t), \quad \rho_1 \psi_t(l, t) = \rho_2 a_2^2 u_x(l, t); \quad x = l, \quad t > 0$$

Here

$$u = w + \frac{P_0(x - l)}{\rho_2 a_2^2} \tag{2}$$

†*Prikl. Mat. Mekh.* Vol. 67, No. 3, pp. 503–511, 2003.

Where a_2 and a_2 are the speed of sound in the liquid and in the material of the obstruction respectively, and $w(x, t)$ is the displacement of points of the obstacle.

The use of the required unknowns ψ and u, whose natures are physically different, for the left- and right-hand layers respectively is due to the fact that the layer $0 \leqslant x \leqslant l$ comprises liquid, and it is convenient to describe its dynamics by means of the velocity potential ψ [3], while the layer $l \leqslant x \leqslant l + \alpha l$ is a solid elastic body, and it is more traditional to formulate the equations in displacements [4] to describe it. Therefore, for hydroelasticity problems, use is often made of such mathematical formulations where the motions of liquid and solid media are described by means of different physical quantities. Here, since the total pressure in the liquid is equal to $P_0 - \rho_1\psi_t$, the function $f(t)$ comprises the difference between the total pressure and the atmospheric pressure at the boundary $x = 0$, and also, owing to representation (2) for displacements of the obstruction when formulating boundary-value problem (1), the constant P_0 is eliminated (does not occur), and furthermore all the initial conditions and the boundary condition at $x = l + \alpha l$ are transformed into homogeneous conditions. It must be pointed out that the right-hand boundary of the liquid has the Euler coordinate $x = l$ only when $t \leqslant \frac{l}{a_1}$, and therefore the contact conditions in (1) are specified approximately when $t > \frac{l}{a_1}$.

Since boundary-value problem (1) is linear, it is of interest to construct an influence function for the disturbances of pressure waves in the liquid in the vicinity of a single discontinuity after the nth reflection from the interface of the media $x = l$. Therefore, the variation with time of the external load $f(t)$ was specified in the form of the Heaviside unit step function

$$f(t) = H(t) \tag{3}$$

having a discontinuity of the first kind at the point $t = 0$. With such a load function, the wave problem (1) will have a solution with strong discontinuities positioned on the characteristics $x + a_nt = \text{const}$ ($n = 1, 2$), where, on the characteristics of the discontinuities, conditions of compatibility will be satisfied [1, 2].

Investigation of problem (1) using elementary theory of acoustic waves [2] is inconvenient, since the case

$$\max\left(\alpha; \frac{\alpha a_1}{a_2}\right) \ll 1 \tag{4}$$

is being considered and, within the time of a single passage of the wave through the liquid layer, a number of reflections will occur in the plate. Therefore, to solve contact problem (1), the method of integral transformations is preferable. Following the well-known approach [2], we will use a complex Fourier transformation with respect to time, defined by the formulae

$$g^F(x, \omega) = \int_0^{+\infty} g(x, t)e^{it\omega}dt, \quad g(x, t) = \frac{1}{2\pi}\int_L g^F(x, \omega)e^{-it\omega}d\omega \tag{5}$$

In the inversion formula, one can choose, as L any straight line parallel to the real axis and having an imaginary part greater than the growth exponent of the function $g^F(x, \omega)$ in the upper complex ω half-plane above the line L [5]. Here, the growth exponent is understood to be the exact lower bound of the set of non-negative numbers $\Omega(\beta)$ such that, for all ω satisfying the condition $\text{Im}\,\omega > \beta$, the limit

$$|g^F(x, \omega)| \leq M|\exp(-i\omega\Omega)|$$

is valid. Here, $M(\beta, \Omega)$ and β are certain positive constants.

For the transforms of the required functions of boundary-value problem (1), taking into account the conditions of compatibility, it is possible to obtain, as previously [2], the ordinary differential equations

$$\psi^F_{xx} + \left(\frac{\omega}{a_1}\right)^2\psi^F = 0, \quad u^F_{xx} + \left(\frac{\omega}{a_2}\right)^2 u^F = 0 \tag{6}$$

with the general solution

$$\psi^F(x,\omega) = A_1(\omega)\sin\left(\frac{x\omega}{a_1}\right) + B_1(\omega)\cos\left(\frac{x\omega}{a_1}\right)$$
$$u^F(x,\omega) = A_2(\omega)\sin\left(\frac{x\omega}{a_2}\right) + B_2(\omega)\cos\left(\frac{x\omega}{a_2}\right) \tag{7}$$

To determine the unknowns A_1, B_1, A_2, and B_2, which are functions of ω, use is made of the boundary and contact conditions of problem (1), written for the transforms. Taking into account expression (3), the equality

$$H^F(\omega) = \frac{i}{\omega} \tag{8}$$

and also the laws of differentiation of the originals and transforms [2, 5], we obtain the system

$$\rho_1 B_1 = \omega^{-2}, \quad A_2 = B_2 \mathrm{tg}\left[\frac{(l+\alpha l)\omega}{a_2}\right]$$
$$B_1\sin\left(\frac{l\omega}{a_1}\right) - A_1\cos\left(\frac{l\omega}{a_1}\right) = ia_1\left[A_2\sin\left(\frac{l\omega}{a_2}\right) + B_2\cos\left(\frac{l\omega}{a_2}\right)\right] \tag{9}$$
$$i\rho_1\left[A_1\sin\left(\frac{l\omega}{a_1}\right) + B_1\cos\left(\frac{l\omega}{a_1}\right)\right] = \rho_2 a_2\left[B_2\sin\left(\frac{l\omega}{a_2}\right) - A_2\cos\left(\frac{l\omega}{a_2}\right)\right]$$

the solution of which yields

$$A_1(\omega) = i(1-\lambda q - \lambda Q + qQ)Z\rho_1^{-1}\omega^{-2}, \quad B_1(\omega) = \rho_1^{-1}\omega^{-2}$$
$$A_2(\omega) = i(\lambda-1)(1-Qr)Z\mu\rho_1^{-1}a_1^{-1}\omega^{-2}, \quad B_2(\omega) = (\lambda-1)(1+Qr)Z\mu\rho_1^{-1}a_1^{-1}\omega^{-2} \tag{10}$$

where

$$q = \exp\left(\frac{2il\omega}{a_1}\right), \quad Q = \exp\left(\frac{2i\alpha l\omega}{a_2}\right), \quad r = \exp\left(\frac{2il\omega}{a_2}\right)$$
$$\mu = \exp[il\omega(a_1^{-1} - a_2^{-1})], \quad \lambda = \frac{(1-K)}{(1+K)}, \quad K = \frac{\rho_1 a_1}{\rho_2 a_2}$$
$$Z = \frac{1}{(1+\lambda q - \lambda Q - qQ)}$$

Hence, the complete solution of boundary-value problem (1)–(3) in transforms is given by formulae (7) and (10). Using well-known theorems of differentiation of transforms and originals [2, 5], we obtain the transforms of the velocities and pressures for the liquid layer, and also the velocities and strains (or stresses) for the plate. The case $\lambda = -1$ corresponds to free surface on the boundary $x = l$, the case $\lambda = 0$ corresponds to the equality of the acoustic impedances of liquid and the obstruction, and the case $\lambda = 1$ corresponds to an obstruction with an infinite impedance; the case $\lambda = -1$ has been well studied [2], and the case $\lambda = 1$ has been previously examined in detail in [1].

It can be shown (in [1] this was shown for the case $\lambda = 1$) that, for a thin obstruction, when condition (4) is satisfied, disturbances of the wave fields in the liquid layer, composed with the case $\alpha = 0$ for any finite $t > \frac{l}{a_1}$, will occur only in small neighbourhood of the liquid behind the discontinuous front of the reflected wave. In order to single out only these disturbances explicitly, it is convenient below to work in all cases with the function

$$\Psi = \psi - \psi_0 \tag{11}$$

where ψ_0 is the solution for the case $\alpha = 0$. For ψ_0 we have the well-known solution

$$\psi_0^F = \frac{\left[\cos\left(\frac{x\omega}{a_1}\right) + i(1+q)(1-q)^{-1}\sin\left(\frac{x\omega}{a_1}\right)\right]}{(\rho_1\omega^2)} \tag{12}$$

which is obtained from the solution (7), (10) with $\lambda = -1$. The disturbance of the pressure in the liquid, as is well-known [1, 3], is equal to

$$P_a = -\rho_1 \Psi_t \tag{13}$$

Then, from relations (7)–(13) we obtain the transform of the influence function of the disturbances of the pressure wave patterns for small α

$$P_a^F(x, \omega) = \frac{-i\theta(1+\lambda)\left(e^{\frac{ix\omega}{a_1}} - e^{\frac{-ix\omega}{a_1}}\right)}{\omega} \tag{14}$$
$$\theta = \frac{q(1-Q)}{[(1-q)(1-qQ+\lambda q-\lambda Q)]}$$

Inverting this expression using formula (5), we have

$$P_a(x, t) = -i\frac{1+\lambda}{2\pi}\int_L \theta(\omega)e^{-it\omega}\left(e^{\frac{ix\omega}{a_1}} - e^{\frac{-ix\omega}{a_1}}\right)\frac{d\omega}{\omega} \tag{15}$$

Expression (15) is inconvenient for further analysis since the integrand has an infinite set of poles and, furthermore, the precise analytical definition of these poles is fairly difficult. Therefore, we will proceed as follows: for cases of a finite impedance of the obstruction ($0 < K < -\infty$; $|\lambda| < 1$) it is always possible to fix the finite value $\mathrm{Im}\ \omega = \beta_1 > 0$, which is independent of α, from the condition

$$|q| < \frac{(1-|\lambda|)}{(1+|\lambda|)} < 1 \tag{16}$$

It is not difficult to verify that, if this condition is satisfied, then

$$|qQ| + |\lambda q| + |\lambda Q| < 1 \tag{17}$$

and thus θ can be expanded in an absolutely convergent series in non-negative powers of $q^n Q^m$

$$\theta = \sum_{n=1,\, m=0}^{\infty} (1-Q)q^n(qQ+\lambda Q-\lambda q)^m = \sum_{n=1,\, m=0}^{\infty} D_{nm}q^n Q^m \tag{18}$$

It can be shown that, with such selection of β_1, the quantity θ will be founded in the half-plane $\mathrm{Im}\ \omega > \beta_1$, and the growth exponent of the function $P_a^F(x, \omega)$ in this half-plane will be equal to zero. Substituting expression (18) into Eq. (15), we will have

$$P_a(x, t) = -i(1+\lambda)\sum_{n=1,\, m=0}^{\infty} D_{nm}\left[I\left(t - d_{nm} - \frac{x}{a_1}\right) - I\left(t - d_{nm} + \frac{x}{a_1}\right)\right] \tag{19}$$

$$I(\tau) = \frac{1}{2\pi}\int_L e^{-i\omega\tau}\frac{d\omega}{\omega}, \quad d_{nm} = 2l\left(\frac{n}{a_1} + \frac{m\alpha}{a_2}\right)$$

where, in order to satisfy the conditions of the inversion theorem (the second expression of (5)) [6], as the contour L we select the straight line $\mathrm{Im}\ \omega = \text{const}$, which satisfies condition (16). The integrals in Eq. (19) can be evaluated immediately using the second expression of (5) and Eq. (8). We obtain

$$I(\tau) = -iH(\tau) \tag{20}$$

Hence it follows that, for any finite t and α in the double series (19), only a finite number of terms will be non-zero, namely those that satisfy the condition

$$d_{nm} - \frac{x}{a_1} < t < d_{nm} + \frac{x}{a_1} \tag{21}$$

Let us consider the situation after the jth reflection of the wave from the contact boundary $x = l$

$$t = \frac{[(2j-1)+\varsigma]l}{a_1}, \quad x = (1-\varsigma)l + \xi; \quad 0<\varsigma<1, \quad 0<\xi<l\varsigma \tag{22}$$

Substituting expressions (22) into condition (21) we obtain the inequality

$$n - \frac{\xi}{(2l)} < j - \frac{ma_1\alpha}{a_2} < n + 1 - \varsigma + \frac{\xi}{(2l)} \tag{23}$$

which can be satisfied only by the values $1 \leqslant n \leqslant j$. Owing to the factor $(1 - Q)$ in expression (14) and the piecewise constancy of integral (20), most terms in series (19) with the same subscript n will also cancel each other out.

From expressions (19) and (20) it follows that the disturbance of the pressure field in the liquid will always have the form of a multistep piecewise-constant function. For small α, these functions in a number of cases can be approximated by continuous functions to which the asymptotic form at $\alpha \to +0$ reduces. We will examine this asymptotic form. Let

$$R_j^e(\eta) = P_a(x,t), \quad \eta = \frac{a_2\xi}{(2\alpha la_1)} \tag{24}$$

where x and t satisfy relations (22). Omitting the details of the proof, we can propose the following algorithm for the successive calculation of $R_j^e(\eta)$.

We will represent the ordinary series in the form of a double series

$$\sum_{k=0}^{\infty} (Q\lambda + qQ - q\lambda)^k = \sum_{n=0,\, m=0}^{\infty} S(n,m)q^nQ^m \tag{25}$$

where the expressions $S(n, m)$ must always be written in the following way

$$S(n,m) = \begin{cases} \lambda^m T(n,m,\lambda), & \lambda > 0 \\ (-\lambda)^m T(n,m,\lambda)\cos\pi m, & \lambda < 0 \\ H(n-m) - H(n-m-1), & \lambda = 0 \end{cases} \tag{26}$$

The approximate asymptotic values of the wave disturbances $R_j(\eta)$ are defined by such a scheme

$$R_0(\eta) = 0, \quad R_{j+1}(\eta) = R_j(\eta) + (1+\lambda)S(j,\eta), \quad j \geq 0 \tag{27}$$

Disturbances of the pressure field in the liquid when $\lambda \neq 0$ after the first three reflections from the plate, calculated by means of algorithm (25)–(27), take the form

$$R_1(\eta) = (1+\lambda)\varphi(\eta,\lambda), \quad R_2(\eta) = (\lambda^{-1} - \lambda)[(1+\lambda)\eta + \lambda]\varphi(\eta,\lambda)$$

$$R_3(\eta) = (1+\lambda)\left[\frac{1}{2}\eta^2(\lambda^{-1} - \lambda)^2 + \eta\left(\lambda^{-1} - 1 - \lambda + \frac{3}{2}\lambda^2 - \frac{1}{2}\lambda^{-2}\right) + 1 - \lambda + \lambda^2\right]\varphi(\eta,\lambda) \tag{28}$$

$$\varphi(\eta,\lambda) = \frac{(\lambda\,\mathrm{sign}\lambda)^\eta[1 + \mathrm{sign}\lambda + (1 - \mathrm{sign}\lambda)\cos\pi\eta]}{2}$$

Exact solutions $R_j^e(\eta)$ are piecewise-constant multistep functions and are expressed in terms of $R_j(\eta)$ as follows:

$$R_j^e(\eta) = R_j([\eta]) \tag{29}$$

where $[\eta]$ is the integer part of the non-negative number η. An analysis of several first asymptotic forms (27) and (28) showed that the maximum error from approximating the exact discontinuous solutions (29) by smooth continuous functions (27) and (28) is of order $1 - |\lambda|$ when $(1 - |\lambda|) \ll 1$, and of order $(|\lambda| \ln |\lambda|)^{-1}$ when $0 < |\lambda| \ll 1$. Thus, approximations (27) and (28) are extremely accurate in cases where the acoustic impedances of the layers differ considerably (by an order of magnitude). However, if the wave impedances of the layers are similar or of the same order, it is recommended that accurate solutions be constructed from formulae (29), because the error of approximation for small λ may be extremely large and, moreover, may approach infinity as $\lambda \to 0$.

For equal impedances we have the trivial situation

$$K = 1, \quad \lambda = 0, \quad R_j(\eta) = H(j-1-\eta), \quad R_j^e(\eta) = H(j-\eta)$$

It can be shown that taking the limit as $a_2 \to +\infty$ reduces expressions (28) to asymptotic forms for an obstruction with infinite impedance. These asymptotic forms have been investigated in detail in [1] and, to find them, the convenient recurrence formula

$$R_j^0(\chi) = 2e^{-\chi}G_j(\chi), \quad j \geq 1, \quad \chi = \frac{\rho_1\xi}{(\rho_2\alpha l)}$$

$$G_1(\chi) = 1, \quad G_{j+1}(\chi) = 1 - G_j(\chi) + 2\int_0^{\chi} G_j(\chi')d\chi' \tag{30}$$

has been obtained, making it possible to calculate rapidly the pressure disturbances after a large number of reflections from the film. For the second and third reflection, we have

$$G_2(\chi) = 2\chi, \quad G_3(\chi) = 2\chi^2 - 2\chi + 1$$

An analysis of expressions (28) indicates that the asymptotic forms as functions of the dimensionless coordinate $\xi \frac{\xi}{(\alpha l)}$ depend on two other dimensionless parameters, $\frac{a_1}{a_2}$ and $\frac{\rho_1}{\rho_1}$ and do not depend on the thickness of the plate. As α decreases, they will contract in proportion like "bellows" towards the discontinuous front of the reflected were along the x coordinate, in the case keeping all their maxima and minima constant, and also the value at the point of discontinuity $\xi = +0$. Therefore, from the mathematical viewpoint, the quantities (28) and (30) comprise the convergence defect of the first derivatives of solutions with strong discontinuities of the boundary-value contact hyperbolic problem (1) in the asymptotic form $\alpha \to +0$. As functions of η, they depend only on one other parameter. The "liquid-obstruction" acoustic impedance ratio.

For obstructions with a high wave impedance $K < 1$, $\lambda > 0$, and pressure disturbances decay exponentially with distance from the discontinuous front. For obstructions with a low impedance $K > 1$, $\lambda < 0$, and the disturbances decay exponentially with oscillations, the period of which along the x coordinate is equal to $\frac{4a_1\alpha l}{a_2}$. In the case of equal impedances ($K = 1$, $\lambda = 0$) or zero impedance of the obstruction ($\rho_2 = 0$ and/or $a_2 = 0$, $K = +\infty$, $\lambda = -1$), there will be no disturbances.

As already mentioned, in the case of similar impedances ($K \approx 1$), formulae (27) and (28) may lose their accuracy. A more detailed analysis showed that a considerable loss of accuracy is observed in the range $0 < \eta < j$. Consider, as an example, the asymptotic form $R_2(\eta)$ when $K = 1 - \varepsilon$; $\varepsilon > 0$. It can be shown that, within the range $0 < \eta < 1$, this function has an extremum approaching infinity as $\varepsilon \to +0$, while the accurate solution is constant and bounded in this range. Therefore, for K close to unity, the asymptotic approximate formulae (27) can be used when $\eta \geqslant j$, and when $0 \leqslant \eta < j$ it is necessary to construct piecewise-constant functions (29). Proceeding in this way in the range $0 \leqslant n < j$, we obtain a step function close to unity, that must be regarded not as a disturbance but rather as the natural continuation of the incident wave, caused by elongation of the calculated liquid layer by a small amount proportional to the thickness of the obstruction. For equal impedances ($K = 1$, $\lambda = 0$), the asymptotic forms and accurate solutions are equal to $H(j - 1 - \eta)$ and $H(j - \eta)$ respectively, and are not disturbances. From the physical viewpoint, the case $K \approx 1$ is of no interest. The use of ratio (29)

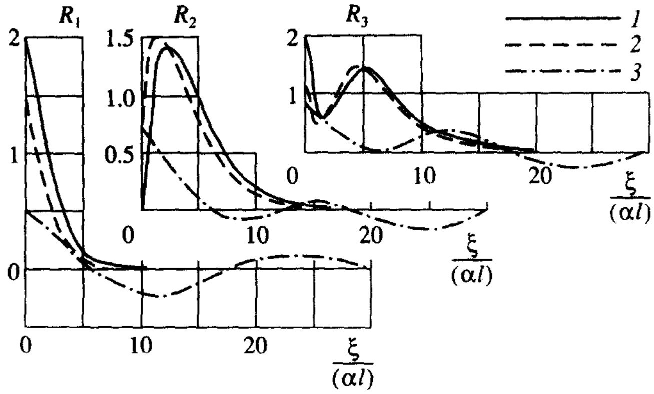

Fig. 1

in the range $0 < \eta < j$ is also recommended in situations where the impedances of the layers are different but have the same order of magnitude ($|\lambda| = O(1)$, $1 - |\lambda| = O(1)$), since the error of approximation in these cases may be of the order of unity for non-integer values of the argument.

Figure 1 shows the pressure disturbances (28) after the first three reflections from the plate for $\frac{\rho_1}{\rho_2} = \frac{1}{2}$ and different values of $\sigma = \frac{a_1}{a_2}$. Curves 1, 2 and 3 are drawn for the following values

$$\sigma = 0\ (K = 0, \lambda = 1), \sigma = \frac{2}{3}\ \left(K = \frac{1}{3}, \lambda = \frac{1}{2}\right), \sigma = 6\ \left(K = 3, \lambda = -\frac{1}{2}\right).$$

To provide more information, the graphs are plotted in independent dimensionless coordinates $R_j\left(\frac{\xi}{(\alpha l)}\right)$, since the argument η in formula (24) is itself a function of σ. Note that $\sigma = 0$ corresponds to an obstruction with an infinite impedance, $\sigma = 2$ corresponds to equal impedances of the liquid and obstacle, and $\sigma = +\infty$ corresponds to the free surface at the boundary $x = l$.

It has been shown [1] that, in the case of an obstruction with infinite impedance, all disturbances with an even number j of reflection from a film have a zero at the point $\xi = +0$. An analysis of expressions (27) and (28) indicates that $a_2 = +\infty$ is the only case where even asymptotic forms are zero at this point, with the exception of the trivial situations of a free surface on the right ($a_2 = 0$ or $\rho_2 = 0$) or equal acoustic impedances, when there are no disturbances of the wave patterns at all.

Considering functions (27)–(29) as influence functions of pressure disturbances at a single discontinuity, it is possible to put forward a simple approximate method for constructing pressure wave patterns for cases of an arbitrary load $f(t)$ having a finite set (or denumerable set without crowding points) of discontinuities of the first kind in boundary-value problem (1) and thin obstructions. The pressure in the liquid must be represented in the form of a linear superposition of the basic and disturbed components. The basic component is the well-known solution for the case when these is no sheet (the right-hand surface of the liquid is free, $\alpha = 0$), and the disturbed component is the function (27) or (29) multiplied by the corresponding magnitude of the discontinuity and rapidly attenuating with distance form the discontinuity front.

It must be pointed out that, for contact problems of the "liquid–liquid" or "solid–solid" type, the influence function for wave disturbances (15) will not vary (for a solid it will only change sign and will be a stress disturbance). Therefore, the wave disturbances investigated here will occur in the case of any contact wave problems with strong discontinuities and thin layers: hydroelasticity, hydromechanics, and the theory of elasticity. As already pointed out above, these wave field disturbances are singular, and they do not vanish when $\alpha \to +0$: the wave pattern in the neighbourhood of a discontinuity as $\alpha \to +0$ non-uniformly approaches the wave pattern with $\alpha = 0$. The extremal values of the pressure surges in this case depend only on the parameter K and the number of the reflection, do not depend on α, and may be very considerable (reaching up to twice the size of the discontinuity, see Fig. 1). For continuous load functions, these effects are not observed. If $f(t) \in C$, the solutions of boundary-value problem (1) will either be classical (if $f(t) \in C_1$) or have weak discontinuities. Such solutions with their first derivatives as $\alpha \to +0$ uniformly approach the corresponding solutions with $\alpha = 0$ in any finite rectangle $0 \leqslant x \leqslant l$; $0 \leqslant t \leqslant t_* < +\infty$.

The results (27)–(29) can be used in methods of acoustic on-destructive testing of the quality of articles with thin coatings (such as painted or spray-coated surface, nickel plating, chromium plating, etc.). Assuming the physicomechanical characteristics of the base layer to be known, by loading the specimen with an added thin surface layer and applying a discontinuous pulse, and recording the reflected signals, it is possible clearly to determine (a) the impedance of the coating from the magnitude of the disturbance surges and (b) the ratio of the coating thickness to the velocity of sound in the coating from the degree of attenuation of the disturbances.

However, in this case, problems of the technological nature may arise because of the difficulty of generating a signal that, for an ultrathin film, can be considered to be discontinuous: the rise time of such a signal should be of a higher order of smallness than the ratio $\frac{\alpha l}{a_2}$.

REFERENCES

1. ROMASHCHENKO, V. A., Anomalous reflection of non-classical waves from ultrathin obstacles. *Prikl. Mat. Mekh.*, 2000, **64**, 6, 1027–1032.
2. ZAHRII, O. Yu. and ULITKO, A. F., *Introduction to the Mechanics of Non-stationary Oscillations and Waves*. Vyshcha Shkola, Kiev, 1989.
3. LOITSYANSKII, L. G., *Fluid Mechanics*. Nauka, Moscow, 1970.
4. LANDAU, L. and LIFSHITZ, E., *Course of Theoretical Physics*. Vol. 7. *Elasticity Theory*. Pergamon, Oxford, 1970.
5. SVESHNIKOV, A. G. and TIKHONOV, A. N., *The Theory of Functions of a Complex Variable*. Nauka, Moscow, 1970.

Translated by P.S.C.